

DESCRIPTION OF DIFFERENT PROPERTIES OF
LIQUIDS BY IDENTICAL FORMULAS

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UDC 536.22:519.24

Using the example of ordinary water, we show that four properties — λ , η , u , and c_p — of liquids can be described by identical formulas over a wide range of temperatures and pressures.

Starting from the possibility of using Tait's isotherm equation

$$v = v_0 \left[1 - A \ln \frac{p + B}{p_0 + B} \right] \quad (1)$$

to determine the density ρ and an identical formula to determine the thermal conductivity λ of liquids, it was established in [1] that the relation between λ and ρ at a given temperature and different pressures can be expressed by the formula

$$\frac{\lambda}{\lambda_0} = \left(\frac{\rho}{\rho_0} \right)^{z_\lambda} \quad (2)$$

or

$$\lambda = B_\lambda \rho^{z_\lambda} = \lambda_s \left(\frac{\rho}{\rho_s} \right)^{z_\lambda}, \quad (3)$$

where

$$B_\lambda = \frac{\lambda_0}{\rho_0^{z_\lambda}} = \frac{\lambda_s}{\rho_s^{z_\lambda}} = \frac{\lambda}{\rho^{z_\lambda}} \left[\frac{\text{W/m}\cdot\text{K}}{(\text{g/cm}^3)^{z_\lambda}} \right]. \quad (3')$$

When there is a dimensionless quantity z_λ , the coefficient B_λ for a given temperature can, as is clear from (3'), be determined by parameters — the reference values (ρ_0, λ_0) , the saturation (ρ_s, λ_s) , or any point (ρ, λ) .

In Fig. 1 we show the possibility of using formula (3) for determining the thermal conductivity of ordinary water, since, according to the data of [2, 3], $\log \lambda$ as a function of $\log \rho$, at a specified temperature and different pressures, varies according to the linear law

$$\lg \lambda = \lg B_\lambda + z_\lambda \lg \rho. \quad (4)$$

Here the quantities z_λ and B_λ will have particular values for each temperature.

The exponent z_λ in formula (2) can be found from two points lying on a given isotherm for different pressures, or by the least-squares method from two equations set up, in accordance with (4), for n experimental points. The values of z_λ , B_λ , and λ_s found in this way for ordinary water, using the given values of λ [2] and ρ [3] for a number of temperatures, are shown in Table 1.

Our investigations [4, 5] showed that in addition to calculating the thermal conductivity, an equation identical with Tait's isotherm equation can be used for calculating a number of thermophysical parameters — the dynamic viscosity, the velocity of sound in the liquid, and the isobaric heat capacity. Then the formulas for the above thermophysical quantities can be given by the following expressions, respectively:

$$\eta = B_\eta \rho^{z_\eta} = \eta_s \left(\frac{\rho}{\rho_s} \right)^{z_\eta}, \quad (5)$$

$$u = B_u \rho^{z_u} = u_s \left(\frac{\rho}{\rho_s} \right)^{z_u}, \quad (6)$$

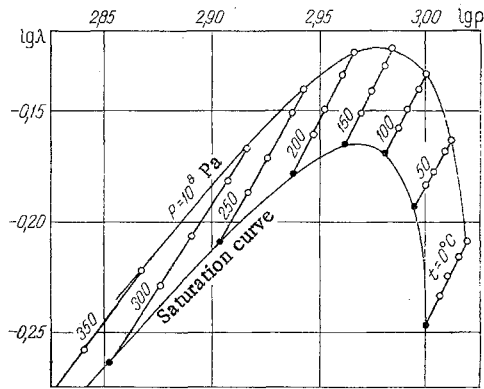


Fig. 1. Graph of $\log \lambda = f(\log \rho)$ for ordinary water. λ , W/m · K; ρ , kg/m³.

TABLE 1. Values of z_λ , $B_\lambda \left[\frac{\text{W/m} \cdot \text{K}}{(\text{g/cm}^3)^{z_\lambda}} \right]$, λ_s (W/m · K); z_η , $B_\eta \left[\frac{\mu\text{Pa} \cdot \text{sec}}{(\text{g/cm}^3)^{z_\eta}} \right]$, η_s ($\mu\text{Pa} \cdot \text{sec}$); z_u , $B_u \left[\frac{\text{m/sec}}{(\text{g/cm}^3)^{z_u}} \right]$, u_s (m/sec); z_{c_p} , $B_{c_p} \left[\frac{\text{J/g} \cdot \text{K}}{(\text{g/cm}^3)^{z_{c_p}}} \right]$, c_{p_s} $\left(\frac{\text{J}}{\text{g} \cdot \text{K}} \right)$

for Water in Formulas (3), (5)-(7), Calculated according to the Data of [2, 3, 6, 7]

| t , °C | z_λ | $\frac{B_\lambda}{\lambda_s}$ | z_η | $\frac{B_\eta}{\eta_s}$ | z_u | $\frac{B_u}{u_s}$ | z_{c_p} | $\frac{B_{c_p}}{c_{p_s}}$ |
|----------|-------------|-------------------------------|----------|-------------------------|-------|-------------------|-----------|---------------------------|
| 0 | 1,91 | 0,567 0,567 | -1,70 | 1769,0 1770,0 | 2,47 | 1402 1401 | -2,40 | 4,226 4,228 |
| 50 | 1,72 | 0,654 0,641 | 0,75 | 552,0 547,0 | 2,62 | 1592 1543 | -0,82 | 4,108 4,149 |
| 100 | 1,95 | 0,734 0,675 | 2,13 | 308,3 281,6 | 2,74 | 1738 1547 | -0,83 | 4,038 4,183 |
| 150 | 1,98 | 0,808 0,681 | 2,45 | 225,1 182,0 | 2,83 | 1876 1468 | -0,98 | 3,945 4,295 |
| 200 | 1,84 | 0,864 0,661 | 2,30 | 187,2 134,0 | 2,89 | 2030 1334 | -1,19 | 3,770 4,482 |
| 250 | 1,70 | 0,900 0,615 | 2,00 | 165,7 105,8 | 2,89 | 2206 1154 | -1,41 | 3,521 4,829 |
| 300 | 1,52 | 0,910 0,544 | 1,63 | 150,0 86,3 | 2,87 | 2427 917 | — | — |
| 350 | 1,31 | 0,891 0,431 | 1,36 | 138,1 65,0 | — | — | — | — |

$$c_p = B_{c_p} \rho^{z_{c_p}} = c_{p_s} \left(\frac{\rho}{\rho_s} \right)^{z_{c_p}}. \quad (7)^*$$

The validity of formulas (5)-(7) for ordinary water is confirmed, as in the case of (4), by the linearity of $\log \eta$, $\log u$, and $\log c_p$ as functions of $\log \rho$. Using the dynamic-viscosity data from [6], the velocity of sound in the liquid from [7], and the isobaric heat capacity from [3], we can use the method described above to find the values of z_η , B_η , η_s ; z_u , B_u , u_s ; z_{c_p} , B_{c_p} , c_{p_s} in formulas (5)-(7) for ordinary water for a number of temperature values, shown in Table 1.

*This is somewhat similar to the formula $c_p = c_{p_0} \frac{\alpha}{\alpha_0} \left(\frac{\rho_0}{\rho} \right)^{3,5} \left(\frac{T_{cr} - T}{T_{cr} - T_0} \right)^n = B \rho^{-3,5}$, given for saturated liquid in [8].

From all of the foregoing it follows that since Tait's isotherm equation is applicable to all classes of liquids, their parameters λ , η , u , and c_p , which are dependent on temperature and pressure, must be subject on the isotherms to identical interpolation formulas of the form

$$\Phi(p, T) = B(T) \rho^{z(T)}(p, T) = \Phi_s(T) \left[\frac{\rho(p, T)}{\rho_s(T)} \right]^{z(T)}. \quad (8)$$

In conclusion, it should be noted that the proposed formulas (3), (5)-(7) reproduced the transport properties λ and η of ordinary water within the limits of the assumptions made in [3, 6], from 0 to 350°C and for pressures from p_s to 10^8 Pa. As for the parameters u (6) and c_p (7), we can state that since the linear relations do not hold for $\log u$ as a function of $\log \rho$ above 300°C and for $\log c_p$ as a function of $\log \rho$ above 250°C, for the velocity of sound in the liquid and the isobaric heat capacity we restricted ourselves to limits from 0 to the indicated temperatures, respectively. A check showed that the average error of the proposed formulas (3), (5)-(7) was less than 0.5%.

NOTATION

p , pressure; p_s , saturation pressure; v , specific volume; v_0 , the same for p_0 ; ρ , density; ρ_0 , the same for p_0 ; ρ_s , the same for a saturated liquid; λ , thermal conductivity; λ_0 , the same for p_0 ; λ_s , the same for a saturated liquid; η , dynamic viscosity; η_s , the same for a saturated liquid; u , velocity of sound in the liquid; u_s , the same for a saturated liquid; c_p , isobaric heat capacity; c_{ps} , the same for a saturated liquid; z , B , coefficients.

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